Exercice 1 : (7 pts)

$\widehat{X,Y}(\alpha)$:

$$\begin{cases} x' = cos(\alpha)(x - x_0) - sin(\alpha)(y - y_0) + x_0 \\ y' = sin(\alpha)(x - x_0) + cos(\alpha)(y - y_0) + y_0 \end{cases}$$

$(\overrightarrow{OX}, \overrightarrow{OY}, \overrightarrow{OZ}) \longrightarrow (\overrightarrow{O'X'}, \overrightarrow{O'Y'}, \overrightarrow{O'Z'}) : t(3, 5, 7) + \widehat{X,Z}(90^o) + \widehat{X,Y}(90^o)$ **(1)**

$$\Downarrow$$

$P_{(\overrightarrow{OX},\overrightarrow{OY},\overrightarrow{OZ})} \longrightarrow P_{(\overrightarrow{O'X'},\overrightarrow{O'Y'},\overrightarrow{O'Z'})} : t(-3, -5, -7) + \widehat{X,Z}(-90^o) + \widehat{X,Y}(-90^o)$ **(1)**

$\widehat{X,Z}(\alpha)$:

$$\begin{cases} x' = cos(\alpha)(x - x_0) - sin(\alpha)(z - z_0) + x_0 \\ z' = sin(\alpha)(x - x_0) + cos(\alpha)(z - z_0) + z_0 \end{cases}$$ **(1)**

$\widehat{X,Y}(\alpha)$:

$$\begin{cases} x' = cos(\alpha)(x - x_0) - sin(\alpha)(y - y_0) + x_0 \\ y' = sin(\alpha)(x - x_0) + cos(\alpha)(y - y_0) + y_0 \end{cases}$$ **(1)**

$\widehat{X,Z}(-90^o)$:

$$\begin{cases} x' = z \\ y' = y \\ z' = -x \end{cases}$$ **(1)**

$\widehat{X,Y}(-90^o)$:

$$\begin{cases} x' = y \\ y' = -x \\ z' = z \end{cases}$$ **(1)**

$P(-2, -3, 4) \rightarrow t(-3, -5, -7) \rightarrow (-5, -8, -3) \rightarrow \widehat{X,Z}(-90^o) \rightarrow (-3, -8, 5) \rightarrow \widehat{X,Y}(-90^o) \rightarrow (-8, 3, 5)$ **(1)**

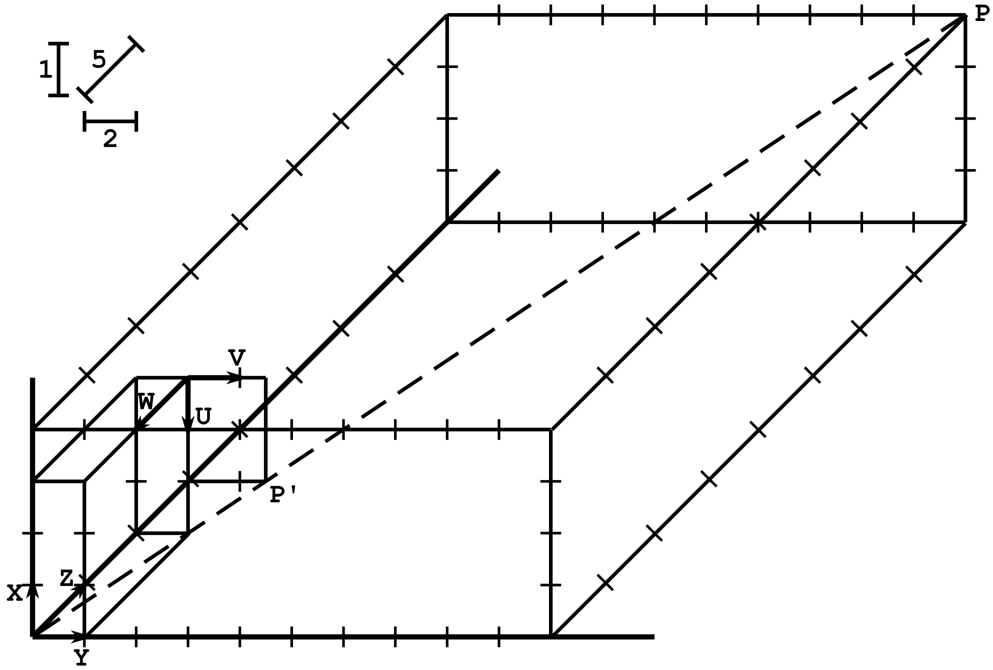
Exercice 2 : (7 pts)

(1) $$\begin{cases} u = -k_u \frac{f}{z} x + u_0 \\ v = k_v \frac{f}{z} y + v_0 \end{cases} \Rightarrow \begin{cases} u = -80 \frac{10}{40} (4) + (240) \\ v = 40 \frac{10}{40} (20) + (-80) \end{cases}$$ **(1)**

$$\Rightarrow \begin{cases} u = (-80) + (240) \\ v = (200) + (-80) \end{cases}$$

$$\Rightarrow \begin{cases} u = (160) \\ v = (120) \end{cases}$$ **(1)**

Schéma : **(2)**



3)

- oui,
- Alignés avec F. **(2)**

Exercice 3 : (6 pts)

Matrice M :

$$M = \begin{pmatrix} -20 & 0 & 10 & -30 \\ 0 & -30 & 40 & 240 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

$r_3 = m_3 = (0, 0, 1)$ **(0.5)**

$$\begin{aligned} u_0 &= m_1.m_3 \\ &= (-20, 0, 10).(0, 0, 1) \\ &= -20 * 0 + 0 * 0 + 10 * 1 \\ &= 0 + 0 + 10 \\ u_0 &= 10 \end{aligned}$$ **(0.5)**

$$\begin{aligned} v_0 &= m2.m3 \\ &= (0, -30, 40).(0, 0, 1) \\ &= 0 * 0 + -30 * 0 + 40 * 1 \\ &= 0 + 0 + 40 \\ v_0 &= 40 \end{aligned}$$ **(0.5)**

$\alpha_u = -\|m_1 \wedge m_3\|$

$$= -\left\| \begin{pmatrix} -20 \\ 0 \\ 10 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\|$$
$$= -\left\| \begin{pmatrix} (0)\times(1)-(0)\times(10) \\ (10)\times(0)-(1)\times(-20) \\ (-20)\times(0)-(0)\times(0) \end{pmatrix} \right\|$$
$$= -\left\| \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix} \right\|$$
$$= -\sqrt{(0)^2+(20)^2+(0)^2}$$

$\alpha_u = -20$ **(0.5)**

$\alpha_v = \|m_2 \wedge m_3\|$

$$= \left\| \begin{pmatrix} 0 \\ -30 \\ 40 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\|$$
$$= \left\| \begin{pmatrix} (-30)\times(1)-(0)\times(40) \\ (40)\times(0)-(1)\times(0) \\ (0)\times(0)-(0)\times(-30) \end{pmatrix} \right\|$$
$$= \left\| \begin{pmatrix} -30 \\ 0 \\ 0 \end{pmatrix} \right\|$$
$$= \sqrt{(-30)^2+(0)^2+(0)^2}$$

$\alpha_v = 30$ **(0.5)**

$r_1 \ = \ (1/\alpha_u)\ [m_1 - u_0 \times m_3]$
$= \ (1/(-20))\ [(-20,0,10) \ - \ 10 \times (0,0,1)]$
$= \ (1/(-20))\ [(-20,0,10) \ - \ ((10)\times(0),(10)\times(0),(10)\times(1))]$
$= \ (1/(-20))\ [(-20,0,10) \ - \ (0,0,10)]$
$= \ (1/(-20))\ ((-20)-(0),(0)-(0),(10)-(10))$
$= \ (1/(-20))\ (-20,0,0)$
$r_1 \ = \ (1,0,0)$ **(0.5)**

$r_2 \ = \ (1/\alpha_v)\ [m_2 - v_0 \times m_3]$
$= \ (1/(30))\ [(0,-30,40) \ - \ 40 \times (0,0,1)]$
$= \ (1/(30))\ [(0,-30,40) \ - \ ((40)\times(0),(40)\times(0),(40)\times(1))]$
$= \ (1/(30))\ [(0,-30,40) \ - \ (0,0,40)]$

$= (1/(30))\,((0)-(0),(-30)\ -\ (0),(40)-(40))$
$= (1/(30))\,(0,-30,0)$
$r_2\ =\ (0,-1,0)$ **(0.5)**

$t_x\ =\ (1/\alpha_u)\,(m_{14}-u_0\times m_{34})$
$= (1/(-20))\,(-30-(10)*(3))$
$= (1/(-20))\,(-30-(30))$
$= (1/(-20))\,(-60)$
$t_x\ =\ 3$ **(0.5)**

$t_y\ =\ (1/\alpha_v)\,(m_{24}-v_0\times m_{34})$
$= (1/(30))\,(240-(40)*(3))$
$= (1/(30))\,(240-(120))$
$= (1/(30))\,(120)$
$t_y\ =\ 4$ **(0.5)**

$t_z\ =\ m_{34}$
$t_z\ =\ 3$ **(0.5)**

Vérification : **(1)**

$$MRes = \begin{pmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_1 & t_x \\ r_2 & t_y \\ r_3 & t_z \end{pmatrix}$$

$$= \begin{pmatrix} -20 & 0 & 10 \\ 0 & 30 & 40 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 4 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

$MRES[0][0]\ =\ (-20)\times(1)+\ (0)*(0)+\ (10)*(0)$
$= (-20)+\ (0)+\ (0)\ =\ -20$
$MRES[0][1]\ =\ (-20)\times(0)+\ (0)*(-1)+\ (10)*(0)$
$= (0)+\ (0)+\ (0)\ =\ 0$
$MRES[0][2]\ =\ (-20)\times(0)+\ (0)*(0)+\ (10)*(1)$
$= (0)+\ (0)+\ (10)\ =\ 10$
$MRES[0][3]\ =\ (-20)\times(3)+\ (0)*(4)+\ (10)*(3)$
$= (-60)+\ (0)+\ (30)\ =\ -30$
$MRES[1][0]\ =\ (0)\times(1)+\ (30)*(0)+\ (40)*(0)$
$= (0)+\ (0)+\ (0)\ =\ 0$
$MRES[1][1]\ =\ (0)\times(0)+\ (30)*(-1)+\ (40)*(0)$
$= (0)+\ (-30)+\ (0)\ =\ -30$
$MRES[1][2]\ =\ (0)\times(0)+\ (30)*(0)+\ (40)*(1)$
$= (0)+\ (0)+\ (40)\ =\ 40$
$MRES[1][3]\ =\ (0)\times(3)+\ (30)*(4)+\ (40)*(3)$
$= (0)+\ (120)+\ (120)\ =\ 240$
$MRES[2][0]\ =\ (0)\times(1)+\ (0)*(0)+\ (1)*(0)$
$= (0)+\ (0)+\ (0)\ =\ 0$
$MRES[2][1]\ =\ (0)\times(0)+\ (0)*(-1)+\ (1)*(0)$
$= (0)+\ (0)+\ (0)\ =\ 0$
$MRES[2][2]\ =\ (0)\times(0)+\ (0)*(0)+\ (1)*(1)$

$$
\begin{aligned}
&= (0) + \ (0) + \ (1) \ = \ 1 \\
MRES[2][3] \ &= \ (0) \times (3) + \ (0) * (4) + \ (1) * (3) \\
&= (0) + \ (0) + \ (3) \ = \ 3
\end{aligned}
$$

$MRES \ = \ M.$