EDPM :

~~exercice (01)~~ ~~(... TD ...)~~

(1)

exercice (02) : $x^2 \frac{\partial^2 u}{\partial x} + 2x \frac{\partial^2 u}{\partial x \partial y} - 3 \frac{\partial^2 u}{\partial y^2} - 5x \frac{\partial u}{\partial x} + 6 \frac{\partial u}{\partial y}$

1) $a = x^2$ (0,25) $b = x$ $c = -3$

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2x \frac{\partial^2 u}{\partial x \partial y} - 3 \frac{\partial^2 u}{\partial y^2} - 5x \frac{\partial u}{\partial x} + 6 \frac{\partial u}{\partial y} = 0$$

$\Delta = b^2 - ac = x^2 + 3x^2 = 4x^2$. (0,25)

hyperbolique (0,5)

2) $\frac{dy}{dx} = \frac{x + 2x}{x^2}$ (0,25) $= \frac{3x}{x^2} = \frac{3}{x}$.

$dy = 3 \frac{dx}{x} \Leftrightarrow y = \ln x^3$ (0,25)

$\Leftrightarrow e^y = c \times x^3$ (0,25) $\Rightarrow \varphi_1 = e^y / x^3$ (0,25)

$\frac{dy}{dx} = \frac{x - 2x}{x^2} = \frac{-1}{x}$ (0,25) $\Leftrightarrow dy = \frac{-dx}{x}$.

$dy = \ln \frac{1}{x}$ (0,25) $\Leftrightarrow e^y = c \times \frac{1}{x}$. (0,25)

$\boxed{\varphi_2 = x e^y}$ (0,25).

(0,5)

$\frac{-3}{x^4} e^y$ ...

e^y ...

$= \frac{-3}{x^3} e^{2y} -$ ...

$\neq$ ...

$\frac{-3x^2}{x^6}$

$$\begin{cases} X = e^y / x^3 \\ Y = x e^y \end{cases} \text{(0,5)} \Leftrightarrow \begin{cases} \frac{\partial X}{\partial x} = \frac{-3}{x^4} e^y , \quad \frac{\partial X}{\partial y} = e^y / x^3 \\ \frac{\partial Y}{\partial x} = e^y , \quad \frac{\partial Y}{\partial y} = x e^y \end{cases}$$ (0,5)

(0,25)

$\frac{\partial u}{\partial x} = \frac{\partial \tilde{u}}{\partial X} \cdot \frac{\partial X}{\partial x} + \frac{\partial \tilde{u}}{\partial Y} \cdot \frac{\partial Y}{\partial x} = \boxed{\frac{-3e^y}{x^4} \frac{\partial \tilde{u}}{\partial X} + e^y \frac{\partial \tilde{u}}{\partial Y}}$ (0,5)

(0,25)

$\frac{\partial u}{\partial y} = \frac{\partial \tilde{u}}{\partial X} \cdot \frac{\partial X}{\partial y} + \frac{\partial \tilde{u}}{\partial Y} \cdot \frac{\partial Y}{\partial y} = \boxed{\frac{e^y}{x^3} \frac{\partial \tilde{u}}{\partial X} + x e^y \frac{\partial \tilde{u}}{\partial Y}}$ (0,5)

$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial x} \right)$ (0,25) $= \frac{\partial}{\partial x} \left(\frac{-3e^y}{x^4} \frac{\partial \tilde{u}}{\partial X} + e^y \frac{\partial \tilde{u}}{\partial Y} \right)$ (0,25) (0,5)

$= \frac{+12 e^y}{x^5} \frac{\partial \tilde{u}}{\partial X} - \frac{3e^y}{x^4} \left[\frac{-3e^y}{x^4} \frac{\partial^2 \tilde{u}}{\partial X^2} + e^y \frac{\partial^2 \tilde{u}}{\partial X \partial Y} \right] + e^y \left[\frac{-3e^y}{x^4} \frac{\partial^2 \tilde{u}}{\partial X \partial Y} + e^y \right.$ ...

$$\boxed{\frac{\partial^2 u}{\partial x^2} = \frac{9e^{2y}}{x^8}\frac{\partial^2 \tilde{u}}{\partial X^2} + e^{2y}\frac{\partial^2 \tilde{u}}{\partial Y^2} - \frac{6e^{2y}}{x^4}\frac{\partial^2 \tilde{u}}{\partial X\partial Y} + \frac{12e^{y}}{x^5}\frac{\partial \tilde{u}}{\partial X}}\quad 0{,}5 \quad \times x^2$$

(2)

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{e^y}{x^3}\frac{\partial \tilde{u}}{\partial X} + xe^y\frac{\partial \tilde{u}}{\partial Y}\right) \quad 0{,}25 \quad 0{,}25$$

$$= \frac{e^y}{x^3}\frac{\partial \tilde{u}}{\partial X} + xe^y\frac{\partial \tilde{u}}{\partial Y} + \frac{e^y}{x^3}\left[\frac{e^y}{x^3}\frac{\partial^2 \tilde{u}}{\partial X^2} + xe^y\frac{\partial^2 \tilde{u}}{\partial X\partial Y}\right] \quad 0{,}5$$

$$+ xe^y\left[\frac{e^y}{x^3}\frac{\partial^2 \tilde{u}}{\partial X\partial Y} + xe^y\frac{\partial^2 \tilde{u}}{\partial Y^2}\right] \quad 0{,}5$$

$$\boxed{\frac{\partial^2 u}{\partial y^2} = \frac{e^{2y}}{x^6}\frac{\partial^2 \tilde{u}}{\partial X^2} + x^2e^{2y}\frac{\partial^2 \tilde{u}}{\partial Y^2} + 2\frac{e^{2y}}{x^2}\frac{\partial^2 \tilde{u}}{\partial X\partial Y} + \frac{e^y}{x^3}\frac{\partial \tilde{u}}{\partial X} + xe^y\frac{\partial \tilde{u}}{\partial Y}} \quad \times(-3)$$

$$\frac{\partial^2 u}{\partial x\partial y} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial x}\left(\frac{e^y}{x^3}\frac{\partial \tilde{u}}{\partial X} + xe^y\frac{\partial \tilde{u}}{\partial Y}\right) \quad 0{,}25 \quad 0{,}25$$

$$= \frac{-3e^y}{x^4}\frac{\partial \tilde{u}}{\partial X} + e^y\frac{\partial \tilde{u}}{\partial Y} + \frac{e^y}{x^3}\left[\frac{-3e^y}{x^4}\frac{\partial^2 \tilde{u}}{\partial X^2} + e^y\frac{\partial^2 \tilde{u}}{\partial X\partial Y}\right] \quad 0{,}5$$

$$+ xe^y\left[\frac{-3e^y}{x^4}\frac{\partial^2 \tilde{u}}{\partial X\partial Y} + e^y\frac{\partial^2 \tilde{u}}{\partial Y^2}\right]$$

$$\boxed{\frac{\partial^2 u}{\partial x\partial y} = \frac{-3e^{2y}}{x^7}\frac{\partial^2 \tilde{u}}{\partial X^2} + xe^{2y}\frac{\partial^2 \tilde{u}}{\partial Y^2} - \frac{2e^{2y}}{x^3}\frac{\partial^2 \tilde{u}}{\partial X\partial Y} - \frac{3e^y}{x^4}\frac{\partial \tilde{u}}{\partial X} + e^y\frac{\partial \tilde{u}}{\partial Y}} \quad \times 2x \quad 0{,}5$$

$$(1) \Leftrightarrow \left[-\frac{6e^{2y}}{x^2} - \frac{6e^{2y}}{x^2} - \frac{4e^{2y}}{x^2}\right]\frac{\partial^2 \tilde{u}}{\partial X\partial Y} + \frac{\partial \tilde{u}}{\partial X}\left[+\frac{12e^y}{x^3} - \frac{3e^y}{x^3} - \frac{6e^y}{x^3}\right]$$

$$+ \frac{\partial \tilde{u}}{\partial Y}\left[2xe^y - 3xe^y\right]$$

$$-16\frac{e^{2y}}{x^2}\frac{\partial^2 \tilde{u}}{\partial X\partial Y} + 3\frac{e^y}{x^3}\frac{\partial \tilde{u}}{\partial X} - xe^y\frac{\partial \tilde{u}}{\partial Y} \quad 0{,}1$$

$$-5\,x \times \left[\frac{-3e^y}{x^3}\frac{\partial \tilde{u}}{\partial X} + e^y\frac{\partial \tilde{u}}{\partial Y}\right] \Leftrightarrow -16\frac{e^{2y}}{x^3}\frac{\partial^2 u}{\partial X\partial Y} + 24\frac{e^y}{x^2} = 0$$

-5

$$+6\times\left[\frac{e^y}{x^3}\frac{\partial \tilde{u}}{\partial X} + xe^y\frac{\partial \tilde{u}}{\partial Y}\right] \Leftrightarrow \boxed{\frac{\partial^2 \tilde{u}}{\partial X\partial Y} = \frac{3xe^{-y}}{2}}$$

$\frac{\partial^2 \tilde{u}}{\partial X\partial Y} =$ [illegible]

$$\Rightarrow u(x,y) = F(x) + G(y) \quad 0{,}5$$

③ alors $\boxed{\frac{\partial^2 \tilde{u}}{\partial x \partial y} = \frac{3}{2}\sqrt{\frac{y}{x}}\,\frac{\partial \tilde{u}}{\partial x}}$

~~$u(x,y) = F\left(e^y/x^3\right) + G\left(x e^y\right)$~~

~~avec F est G de C^1~~ 0,5

exercice (01) 0,75

On a $f = xy^2$, $g = x^2y$, $h = (x^2+y^2)u$

le système caractéristique

$$\frac{dx}{xy^2} = \frac{dy}{x^2y} = \frac{du}{(x^2+y^2)u}$$ 0,5

$$\frac{dx}{xy^2} = \frac{dy}{x^2y} \Leftrightarrow \frac{dx}{y} = \frac{dy}{x}$$ 0,5

$$\Leftrightarrow x\,dx = y\,dy \Leftrightarrow x^2 = y^2 + c$$ 0,5

$$\Rightarrow \boxed{\varphi_1 = x^2 - y^2}$$ 0,25

$$\frac{dy}{x^2y} = \frac{du}{u(x^2+y^2)} \Leftrightarrow \frac{du}{u} = \frac{(x^2+y^2)\,dy}{x^2y}$$ 0,5

$$\Leftrightarrow \frac{du}{u} = \frac{x\,dy}{x^2y} + \frac{y^2\,dy}{x^2y}$$ 0,25

$$= \frac{dy}{y} + \frac{y\,dy}{x^2}$$ 0,25 on a ($y\,dy = x\,dx$) 0,25

$$= \frac{dy}{y} + \frac{x\,dx}{x^2} = \frac{dy}{y} + \frac{dx}{x}$$ 0,25

$$\Leftrightarrow \ln u = \ln y + \ln x + c$$ 0,5

$$\Leftrightarrow \ln u = \ln yx + c$$

$$\Rightarrow C = \frac{u}{xy} \Rightarrow \varphi_2 = \frac{u}{xy}$$ 0,5

Donc $\exists$ F.

$$\Phi = F(\varphi_1, \varphi_2) = F\left(x^2-y^2, \frac{u}{xy}\right)$$ 0,5

$$\Rightarrow \frac{u}{xy} = G(x^2-y^2)$$ 0,5

$$\Leftrightarrow u = xy\,G(x^2-y^2)$$

$$\begin{vmatrix} \nabla\varphi_1 & \nabla\varphi_2 \\ 2x & \frac{-yu}{x^2y^2} \\ -2y & \frac{-xu}{x^2y^2} \\ 0 & \frac{1}{x^2y^2} \end{vmatrix}$$ 0,5

$$\begin{cases} \alpha_1\, 2x + \beta\left(-\frac{yu}{x^2y}\right) = 0 \\ 2y\alpha_1 - \beta\left(\frac{-u}{xy^2}\right) = 0 \Rightarrow \alpha_1 = 0 \end{cases}$$ 0,5

$$\beta / x^2y^2 = 0 \Rightarrow \beta = 0$$

Donc sont indépendants